



Lab Session 4 Introduction to the DC Motor

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Equipment

- Agilent 54600B 100 MHz Digitizing Oscilloscope
- Agilent 33120A Function/Arbitrary Waveform Generator
- Agilent 6632A Power Supply
- Agilent 34401A 6 1/2 Digit DMM
- Agilent 35670A Dynamic Signal Analyzer
- Agilent VEE 5.0 software
- Matlab software (www.mathworks.com)
- Amplifier/Motor/Potentiometer test set (hybrid, built in-house)
- Analog computer: Comdyna GP6

Purpose

Several basic features of control systems can be understood by careful examination of dc motor control as a case study. Moreover, the dc motor is an important component in myriad control systems. The first of three dc motor experiments emphasizes system modeling in the time domain. The motor to be studied is shown in Fig. 4.1, and a functional block diagram is shown in Fig. 4.2.

A diagram describing the motor dynamics is shown in Fig. 4.3. This will be used to develop the motor transfer function.

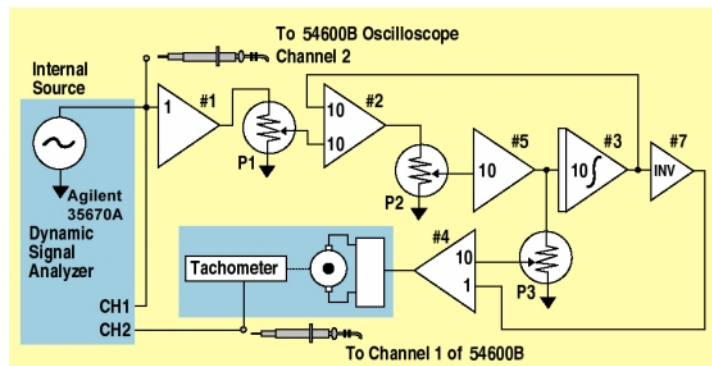


Figure 4. 1. The DC Motor.

The parameters under consideration have the following units: the pot. gain K_{pot} is in volts/rad; the tach. gain K_{tach} is in volts-s/rad.; Armature resistance R_a is measured in ohms; the back emf constant K_V is in volt-s/rad.; the torque constant K_T is in N-m/amp; the total inertia seen by the motor $J = J_M + J_L$ is in Kg-m²; the viscous friction coefficient is b in N-m-sec/rad.; the Coulomb friction coefficient is c in N-m; and the armature inductance L_a is measured in Henries.

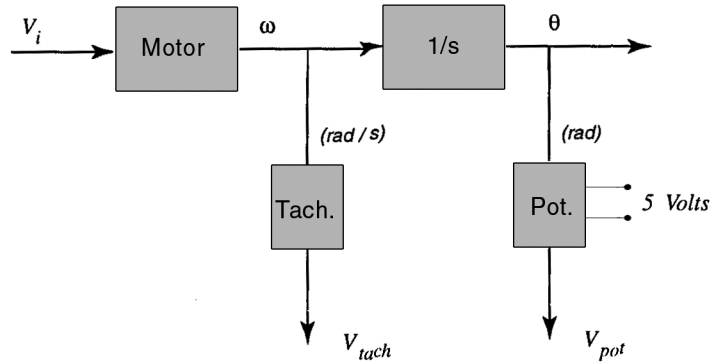


Figure 4.2. Motor Block Diagram.

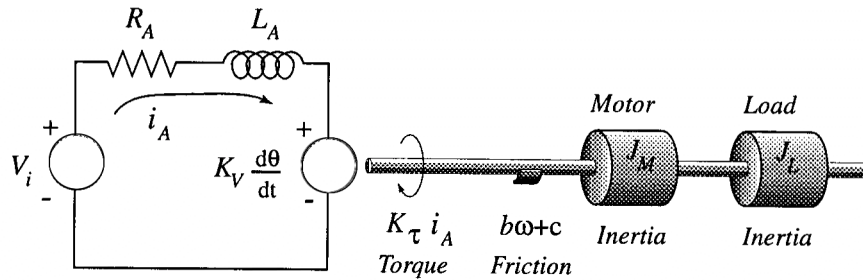


Figure 4.3. Motor Schematic Diagram.

Preparation

Readings:

A basic description of the system components may be found in B. C. Kuo, *Automatic Control Systems*, 7th edition. See Sections 4.5 and 4.6, which cover potentiometers, tachometers and dc motors.

Prelab:

- a. The dc motor, with the flywheel clamped ($\omega = 0$), is described by the electrical schematic of Fig. 4.4. The resistance R_s has been added to obtain voltage measurements, so that the current's time response can be viewed on an oscilloscope. Find $V_o(s)/V_i(s)$ in terms of R_a , R_s , and L_a . What is the electrical time constant, τ_e , for the motor?

Note: The final model of the motor we will develop will not have R_s . This is only used for identification.

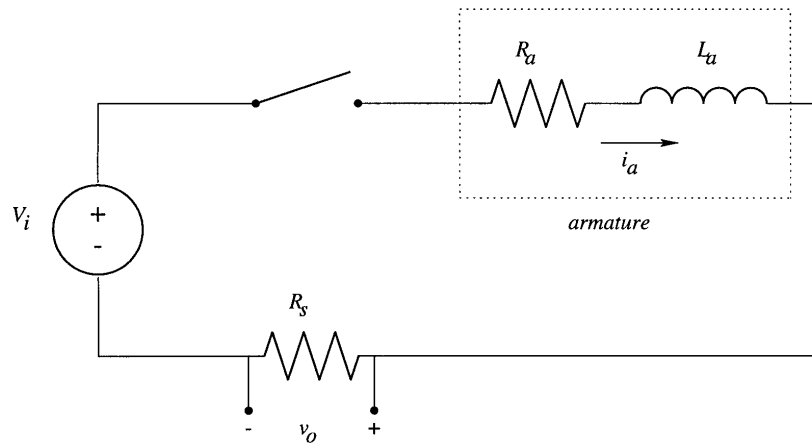


Figure 4.4. Motor Electrical Diagram: Flywheel clamped.

- b. The electrical parameters R_a and L_a (assuming R_s is known) can be determined from the step response of the motor with clamped flywheel. Describe how τ_e can now be found from a best-fit line of $\ln(y_s - y(t))$ vs. t . This will give a reliable estimate in spite of noise. Load the MATLAB data in the file "f:\labs\Ee386\prelab4.mat." The variable "Elect" contains time points and voltage points from a step response in columns 1 and 2, respectively. Determine L_a from these data, after finding τ_{ue} . Assume $R_s = 2.5\Omega$, $R_a = 3.3\Omega$, compare the value you find from the plots to the value used to generate them: 1 mH. (In the Laboratory Exercises below you will collect your own data points.)
- c. Now consider the dc motor with the rotor free to turn. The back emf constant, K_v , the torque constant, K_t ($= K_v$ in SI units), the viscous friction coefficient, b , and the Coulomb friction coefficient, c , can be obtained from *steady-state* step response data, that is from measurements of $\omega(\infty)$ and $i_a(\infty)$, when a known constant input voltage V_i is applied. Explain how (more than one trial will be needed). If we perform two experiments, with following results:

V	Steady state I_a	steady state ω
5 Volts	0.249 A	77.0 rad/s
10 Volts	.304 A	163 rad/s

what are the values for $K_t = K_v$, b and c (R_s is not used in this measurement, R_a and L_a are as above)?

- d. The mechanical time constant, and hence the total moment of inertia, J , can be determined by opening the motor circuit ($I_a = 0$), when the motor is rotating, and observing the time-constant of the resulting decay of the motor speed. The situation is shown in Fig. 4.5.

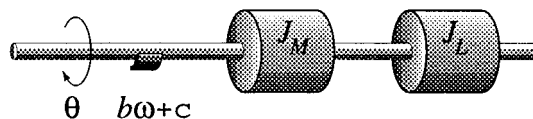


Figure 4.5. Motor Mechanical Diagram: open electrical circuit.



With the motor spinning only in one direction, the Coulomb friction will not change sign, and we can consider it to be a constant. The dynamics are then described by the differential equation:

$$J \dot{\omega} + b\omega + c = 0$$

Find $\omega(t)$, in terms of J , b , c , and $\omega(0)$. Explain how to adjust the method used in part (b) so that we can find J if the other parameters are known. Illustrate your method on the data in the variable "Iner" in the MATLAB data file. Use the values for the friction coefficients from part (c). Compare the value you find to the value used to generate the plots, 4×10^{-5} Kg- m².

- (e) If we assume the motor has a strictly positive angular velocity, we can get a linear model by disregarding the Coulomb friction. (We'll address this again in lab 5.) Using the numerical values above, obtain the resulting second order transfer function, $\Omega(s)/V_i(s)$ for the motor. Compare this transfer function to the first order transfer function obtained in class with $L_a = 0$. Use MATLAB to produce step responses and Bode plots for both cases. Discuss.

Laboratory Exercise

For the identification of the motor you will be collecting a large amount of data. In addition to the data obtained using AgilentVee, the *Data Table* given at the end of this chapter should help you to make sure that you collect all necessary measurements.

Armature Resistance and Inductance:

The armature resistance R_a and inductance L_a are measured by observing a step-response and steady-state behavior, while the rotor is locked. This is done by displaying the voltage drop across a resistor R_s connected in series with the locked motor. The step-by-step instructions are given below:

- Turn on the PC, the scope, DMM, and Agilent power supply (far left).
- Insert your floppy disk in the computer.
- Open the Agilent VEE folder, and double-click the Agilent VEE icon. Use the **File** menu to **Open** file f:/labs/Ee386/386Lab4.vee. You should see the icons for the Agilent scope, and the Agilent 6632A power supply on your screen.
- To initialize the scope, click its object screen. To initialize the power supply, click the reset button.
- Attach the rotor-locking attachment to the motor, and tighten the thumb screws firmly.
- Measure the resistance R_s of the resistor-box (Pomona box with two leads) using the DMM, it should be close to 2.5Ω . Also check the resistance of all the leads you use; the connectors should be tightened if they have more than 0.05Ω resistance.
- On the left-side of the patch panel you should see two jacks labeled Agilent 0-20V. To determine R_a , hook up the black jack directly to the motor's black jack. Hook the red jack of the power supply to the thumb-switch and connect the thumb-switch to the motor's red jack. Using Agilent VEE on the PC, on the power supply, **Set Voltage** should be set at 6 volts and **Set Current** should be set to one amp. With the thumb-switch released, read the voltage from the power supply's LCD screen. This is R_a , and should be between 3 and 5 Ω . Repeat this measurement for two other angles of the locked rotor, and take the smallest of the three values.
- Connect the BNC of the coaxial cable to channel 1 of the scope.
- Disconnect the black lead of the power supply and reconnect it to one end of the resistor-box and connect the other end of the resistor box to the black lead on the motor mount. Connect the black lead of the scope to the end of the resistor box connected to the power supply and the red lead of the scope to the other end of the resistor-box. (Now the circuit is that of Fig. 4.4.)



- Using Agilent VEE on the PC: (i) on the power supply, **Set Voltage** should be set at 5 volts and **Set Current** should be set at 2 amps; (ii) on the scope, **Timebase** should be set to 0.1 ms, and **TRIGLevel** should be 500mV. (You may need to change the V/div setting of the oscilloscope as required.)
- Rotate the flywheel with your fingers to compress to rubber coupling with the rotor locking mechanism. Now turn on the power supply using the **Output Enable** switch on the power supply window. Press and release the thumb-switch until you see you see a good waveform on the scope screen.
- Depress the **Collect Data Displayed on Scope** pushbutton. Each time that you press **Collect Data Displayed on Scope** in Agilent VEE the data from the scope will be stored in the vector $Elect_j$, where the integer j is incremented by the **Counter** each time that you store new data. These vectors are appended sequentially to the file `a:\Lab4.m` on your floppy disk. The waveform is now stored on your disk in coordinate format, so that you can obtain estimates of L_a , outside of lab.
- Turn off the power to the motor using the **Output Enable** switch, to keep it from overheating.
- Repeat this experiment five times to cover the voltage range 5v-10v in 1 volt increments. Change v/div in AgilentVee for the scope if necessary. You will use the technique from the prelab to get an estimate of L_a . Report the average values.

Calibration of the Tachometer:

The potentiometer may be used to obtain accurate estimates of the position Θ of the dc motor. The tachometer, mounted on the motor shaft, produces a voltage that is directly proportional to the angular velocity of the motor shaft. The *gain* of the tachometer expresses the voltage generated by the tachometer per unit angular velocity (rad/sec).

- Remove the rotor-locking attachment and the resistor-box. Connect the power supply directly to the motor. Connect the **Input Hi** of the Agilent 34401A (DMM) to the tachometer's orange lead on the motor mount. The **Input Lo** of the DMM is connected to the tachometer's grey lead.
- Connect the potentiometer to the motor and firmly tighten the thumb screws at the base of the potentiometer mount. Connect the bottom black lead to the **HI +5** volts in the patch panel. Connect the top black lead from the potentiometer mount to the **LO +5** Volts.
- Connect the black lead of the scope to the upper black lead on the potentiometer mount. Connect the red lead of the scope to the center white lead on the potentiometer mount. Press the Measure: **Time** button on the scope itself, and select Period from the menu.
- Turn on the 5 volt power supply connected to the potentiometer using the switch on the patch panel.
- Using the **Set Voltage** switch in the Power Supply window in Agilent VEE, input a voltage of 5 volts to the motor. Using the **Set Current** switch in this window, set the current limit to 2 amperes. Turn the power supply on using the **Output Enable** switch in this window. The motor should begin to spin and you should see a sawtooth waveform on the scope. This is the voltage output by the potentiometer as the motor spins. Using Agilent VEE, adjust the scope **Timebase** and **TRIGLevel** to get two waveforms displayed.
- Record the voltage output by the tachometer on the DMM. This is the voltage generated by the tachometer for the estimated angular velocity of the motor.
- Record the period from the scope window. Now disable the output of the power supply (This saves wear on the potentiometer). We will not need to save the waveform to disk with Agilent VEE.
- The ratio of the voltage output by the tachometer and the angular velocity of the motor measured in radians/second is the tachometer gain. Repeat this experiment twice with the voltage set to, respectively, 10 volts and 15 volts. Present your observations in a



tabular format, and use the average of the three observations as your estimate of the tachometer gain.

Back-EMF Constant and Viscous Friction Coefficient:

To perform this part of the lab you will use the fact that the back-EMF constant K_v and the motor torque constant K_t are equal under the SI system. You will compute for a given applied voltage V_i the resulting steady state armature current I_a , and tach. voltage V_{tach} . From this data you can compute the back-EMF constant $K_v (= K_t)$ using the equation: $V_i = I_a R_a + K_v \omega$.

You can then also compute the viscous- and Coulomb-friction coefficients b and c using the steady state equation: $K_t I_a = b\omega + c$.

The detailed instructions for the measurement of $K_v (= K_t)$ are given below.

- Remove the potentiometer from the motor assembly.
- Within Agilent VEE adjust the **Set Voltage** to 5 volts, and the **Set Current** to 2 amperes.
- Using the **Output Enable** switch on the power supply window turn on the power to the motor. Look at the current drawn by the motor at the right of the front display panel on the power supply. Record the voltage applied V_i across the motor, the steady-state current I_a drawn by the motor observed on the power supply and the voltage generated by the tachometer as observed on the DMM.
- Repeat this experiment 5 times, again applying voltages of 6, 7, 8, 9, and 10. Record your data in tabular form. You will use an average of the estimates of K_v and a best-fit line for b and c in constructing your model.

Rotor Moment of Inertia:

The rotor moment of inertia can be inferred from the mechanical time constant τ_m , and the friction constants b and c . The mechanical time constant can be obtained by observing the decaying angular velocity when the current supply to the motor is cut-off. This is achieved by using a thumb-switch in series with the motor. The detailed steps are given below:

- Connect the power supply to the motor via the thumb-switch (the Pomona box with two leads and an audio jack). The thumb-switch is open when depressed, and is closed otherwise. That is, depressing the thumb-switch will disconnect the power supply from the motor.
- Reset (clear) the **Counter** and change the **Formula** variable name from "Cal" to "Iner".
- Connect the coaxial cable to channel 1 of the scope. Connect the red lead of the coaxial cable to the orange lead of the tachometer, and the black lead to the grey lead of the tachometer. Remove connections to the DMM.
- Using Agilent VEE, set **TRIG Level** to 4 volts, and **Timebase** to 100 ms. Adjust **Set Voltage** to 10 volts.
- You will have to set the scope to trigger on a transition from high to low. Using Agilent VEE, go to the scope trigger panel and change slope to negative. To do this, you need to do the following in Agilent VEE. Click on **Main Panel** of the scope, then select **Trigger Panel**. Then successively pick the menu options **Previous** and **Slope to Negative**.
- Click on **Main Panel** of the scope, then select **Channel Panel**. Set channel one's sensitivity to 1 V/div and offset voltage to 3.5 V.
- Using the **Output Enable** switch in the power supply's window turn on the power to the motor. Wait till the tachometer reading is roughly constant. Depress the thumb switch and *keep it depressed*. Press the **Stop** button on the scope to store the waveform. Turn off the power supply to the motor using the **Output Enable** switch on the power supply's window.
- Depress the **Collect Data Displayed on Scope** pushbutton. The scope waveform is now stored in a coordinate format on your disk. Press **Run** on the scope.



- Repeat this experiment five times, incrementing the voltage 1 V each time to 15 V. You may need to adjust the trigger level to keep the whole trace on the scope.

Report:

1. Explain the process of measuring R_a , L_a , K_v , K_t , b and J . Carefully justify, by electrical and mechanical principles, the experimental (not mathematical) procedures. Include the responses (plots) obtained in the laboratory exercises, *carefully labeled* to show all cases.
2. Report R_a and R_s . Using the slope-method that you applied in the prelab, compute τ_e and L_a .
3. Now examine your individual results from the viscous friction experiment. Plot out the friction torque ($= K_t I_a$) against ω . Determine the values of the Coulomb- and viscous-frictions coefficients, c , and b .
4. Using the graphical method from the prelab, compute J using your estimates of b and c .
5. Explain why K_v and K_t should be identical in the SI system, by applying the *conservation of energy law*. Show also that their units are equivalent.
6. Include the derivation of the motor transfer function. Using the various quantities measured in this experiment compute the linearized transfer function $\Omega(s)/V_i(s)$ of the motor. Compute the poles and zeros of this transfer function. Next derive a first-order model for the transfer function and compute its poles and zeros.
7. For the *nonlinear* model, what will be the steady-state angular-velocity for a 4 V dc input voltage?

Save your results to use in the next experiment.



D.C. DYNAMICS AND MODELING DATA TABLES

The following data is to be collected by hand IN ADDITION to the data to be collected from AgilentVee for MATLAB analysis. Do not forget to indicate units.

1. ARMATURE RESISTANCE AND INDUCTANCE

Resistance, R_s , of resistor box: _____

Resistance, R_a , of armature : _____, Smallest: _____

2. CALIBRATION OF THE TACHOMETER

V_i	Δ_t	Tach. Voltage
5 Volts		
10 Volts		
15 Volts		

Where Δ_t is the time the rotor takes for 2π radians of revolution

3. BACK-EMF CONSTANT AND FRICTION COEFFICIENTS

V_i	steady state I_a	Tach. Voltage
5 Volts		
6 Volts		
7 Volts		
8 Volts		
9 Volts		
10 Volts		